In the np case one of the helicity flip amplitudes, ϕ_{+--+}, causes a secondary maximum to appear at small angles ($\approx 15°$). Although this maximum might be washed out, more experimental data would be valuable to study this feature of the absorption model.

The ratio of the πp to Kp charge-exchange cross sections at the same c.m. momentum, particularly at 0°, can be used to test the SU_3 prediction of coupling constants. However, this test should be carried out at higher energies (e.g., $\gtrsim 6$ BeV) than where data presently exist in order to get outside the πp resonance region. Thus, additional data, especially at higher energies would clearly be of value.

ACKNOWLEDGMENTS

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Electromagnetic Form Factor of the Neutrino*

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Bernstein and Lee and, independently, Meyer and Schiff have recently published calculations of the neutrino electromagnetic form factor, obtaining results differing by a finite constant term. This difference can be traced back to how the W-meson contribution is regularized: The Bernstein-Lee calculation is gauge-invariant at every step, while Meyer and Schiff simply impose over-all neutrino charge neutrality at the end. The ξ-limiting process in addition sums a class of electromagnetic radiative corrections and assigns the value $\ln\alpha^{-1}$ to the logarithmically divergent term in the W-meson contribution. Since the finite term, which is almost comparable to $\ln\alpha^{-1}$ in magnitude, is not fixed by the ξ-limiting method, the neutrino form factor has actually been determined only to order of magnitude by this method. For this reason and because the W mass is large (or infinite), we have determined the largest part of the neutrino form factor from the charged lepton contribution using a guage-invariant direct-interaction theory. This is obtained, without further calculation, from the photon vacuum polarization. The ν_e charge radius thus measures the same integral that appears in the perturbation-theory calculation of $Z_{3e}^{1/2}$, the charge renormalization in quantum electrodynamics.

INTRODUCTION

In the weak interaction theory, either based on the local four-fermion current-current self-interaction (F theory) or on the intermediate boson model (W theory), $(e\nu_e)$ $(e\nu_e)$ and $(\mu\nu_\mu)$ $(\mu\nu_\mu)$ couplings would exist to the lowest order in the weak coupling constant G (F theory) or g^2 (W theory). An immediate consequence of this interaction is that the neutrinos would have electromagnetic interaction through the generation of a charge form factor in the sequence[1]:

$$\text{(i) } F \text{ theory:} \quad \nu_l \rightleftarrows \nu_l + l^+ + l^- \rightleftarrows \nu_l + \gamma\,, \tag{1}$$

$$\text{(ii) } W \text{ theory:} \quad \nu_l \rightleftarrows l^- + W^+ \rightleftarrows l^- + W^+ + \gamma \rightleftarrows \nu_l + \gamma\,, \tag{2}$$

where $l=e$ or μ. The matrix element of the neutrino electromagnetic current operator J_μ evaluated between initial and final one-neutrino states in a γ_5-invariant CP-invariant theory[2] is

$$\langle \nu' | J_\mu | \nu \rangle = i\bar{\nu}(p')\gamma_\mu(1+\gamma_5)\nu(p)F(q^2)\,, \tag{3}$$

where p and p' are, respectively, the initial and final four momenta and $q^2=(p-p')^2$. $F(q^2)$ is the neutrino form factor, which, in lowest order electromagnetic and weak interaction, originates from the Feynman diagrams of Figs. 1 or 2.

The explicit form of $F(q^2)$ has recently been calculated by Bernstein and Lee[3] and independently by Meyer and Schiff[4] in the W theory for the case of vector

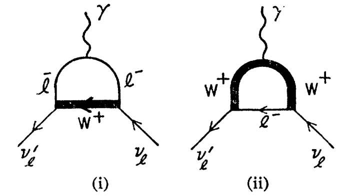

FIG. 1. The lowest order diagrams contributing to the neutrino form factor ($l=e$ or μ) in the W theory.

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[1] In order to satisfy gauge invariance or energy-momentum conservation, the coupling must be to photons off the $q^2=0$ mass shell, i.e., to virtual photons or to plasmons.

[2] We do not consider the question of whether a consistent quantum electrodynamics exists for a massless spinor field.

[3] J. Bernstein and T. D. Lee, Phys. Rev. Letters 11, 512 (1963).

[4] Ph. Meyer and D. Schiff, Phys. Letters 8, 217 (1964).

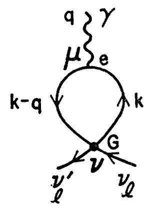

FIG. 2. The lowest order diagram contributing to the neutrino form factor ($l=e$ or μ) in the F theory.

mesons of unit gyromagnetic ratio. Their results for $f(0)$, which is related to the "charge radius" $\langle r^2\rangle$ by the relation $\langle r^2\rangle=(3Ge/8\pi^2\sqrt{2})f(0)$, agree for the term in the mass-singularity and in $\ln\alpha$, but differ by a finite nontrivial constant of the same order as $\ln\alpha$ [cf. $f_{\rm II}(0)$ in columns 2 and 3 of Table I]. This difference traces back to how the W-meson contribution is regularized: The Bernstein-Lee calculation is performed with the gauge-invariant ξ-limiting process; Meyer and Schiff use a Feynman regulator for the lepton propagator in the diagram (ii) of Fig. 1, while imposing the charge neutrality condition

$$F(q^2)=0, \quad \text{at} \quad q^2=0. \qquad (4)$$

In Table I, we also include our own results using Bernstein-Lee and Meyer-Schiff regularization. In column 4 we present the results of using an invariant cutoff Λ on the four-dimensional integration in place of regularization of the integrand.

The W-meson contribution to the neutrino form factor can, however, be determined only to order of magnitude since the limiting value $G(\infty)$,[5] which is dropped in the Bernstein-Lee and Meyer-Schiff calculations, may be comparable to other finite or logarithmic terms re-

TABLE I. Summary of results of calculation of $f(q^2)$, where $\rho=q^2/4m_l^2$. Numerically, $-(5/3)\ln\alpha=8.20$, and with $M=15m_\mu$, we have $\frac{4}{3}\ln(M^2/m_\mu^2)=7.22$, $\frac{4}{3}\ln(M^2/m_e^2)=21.41$. Bernstein and Lee have $f_{\nu_e}(0)=-15.21$, $f_{\nu_\mu}(0)=-1.02$; Meyer and Schiff have $f_{\nu_e}(0)=-15.60$, $f_{\nu_\mu}(0)=-1.41$. Our F-theory (direct current-current interaction) calculation gives, with cutoff $\Lambda=M, f_{\nu_e}(0)=-21.6$, $f_{\nu_\mu}(0)=-7.4$. With a larger value for Λ, there would not occur such a near equality between $f_{\rm I}(0)$ and $f_{\rm II}(0)$ for ν_μ.

Theory	⟨W theory⟩			⟨F theory⟩
Authors (Method used)	Bernstein-Lee (ξ-limiting process)	Meyer-Schiff (Regularization in lepton propagator)	Present calculation (invariant cutoff)	Present calculation[e]
$f_{\rm I}(0)$: [Diagram (i) in Fig. 1]	$-\frac{4}{3}\ln\frac{M^2}{m_l^2}-\frac{2}{9}$[a]	$-\frac{4}{3}\ln\frac{M^2}{m_l^2}-\frac{2}{9}$	$-\frac{4}{3}\ln\frac{M^2}{m_l^2}-\frac{2}{9}$	$-\frac{4}{3}\ln\frac{\Lambda^2}{m_l^2}+\frac{2}{9}$
$f_{\rm II}(0)$: [Diagram (ii) in Fig. 1]	$\frac{5}{3}\ln\frac{\Lambda^2}{M^2}-\frac{16}{9}$[b]; $\frac{5}{3}\ln\frac{\Lambda^2}{M^2}-\frac{7}{6}$[c]	$\frac{5}{3}\ln\frac{\Lambda^2}{M^2}-\frac{13}{6}$; $\frac{5}{3}\ln\frac{\Lambda^2}{M^2}-\frac{5}{4}$[c]	$\frac{5}{3}\ln\frac{\Lambda^2}{M^2}-\frac{5}{6}$	...
$f(0)=f_{\rm I}(0)+f_{\rm II}(0)$	$-\frac{5}{3}\ln\alpha-\frac{4}{3}\ln\frac{M^2}{m_l^2}-2$[d]; $-\frac{5}{3}\ln\alpha-\frac{4}{3}\ln\frac{M^2}{m_l^2}-\frac{25}{18}$[c]	$-\frac{5}{3}\ln\alpha-\frac{4}{3}\ln\frac{M^2}{m_l^2}-\frac{43}{18}$[d]; $-\frac{5}{3}\ln\alpha-\frac{4}{3}\ln\frac{M^2}{m_l^2}-\frac{53}{36}$[c]	$-\frac{5}{3}\ln\alpha-\frac{4}{3}\ln\frac{M^2}{m_l^2}-\frac{19}{18}$[d]	$-\frac{4}{3}\ln\frac{\Lambda^2}{m_l^2}+\frac{2}{9}$
$f_{\nu_e}(0)-f_{\nu_\mu}(0)$	$-\frac{4}{3}\ln\frac{m_\mu^2}{m_e^2}$	$-\frac{4}{3}\ln\frac{m_\mu^2}{m_e^2}$	$-\frac{4}{3}\ln\frac{m_\mu^2}{m_e^2}$	$-\frac{4}{3}\ln\frac{m_\mu^2}{m_e^2}$
$f(q^2)-f(0)$	$-\frac{20}{9}+\frac{4}{3\rho}+\frac{4}{3}\left(1-\frac{1}{2\rho}\right)\left(1+\frac{1}{\rho}\right)^{1/2}\times\ln\frac{(1+1/\rho)^{1/2}+1}{(1+1/\rho)^{1/2}-1}$	...	...	$-\frac{20}{9}+\frac{4}{3\rho}+\frac{4}{3}\left(1-\frac{1}{2\rho}\right)\left(1+\frac{1}{\rho}\right)^{1/2}\times\ln\frac{(1+1/\rho)^{1/2}+1}{(1+1/\rho)^{1/2}-1}$

[a] Bernstein and Lee do not give $f_{\rm I}(0)$ and $f_{\rm II}(0)$ separately, but only $f(0)$ after $(5/3)\ln(\Lambda^2/M^2)$ has been converted into $-(5/3)\ln\alpha$. We have calculated $f_{\rm I}(0)$, which is finite without cutoff, finding agreement with Meyer and Schiff.
[b] Obtained by subtracting above value for $f_{\rm I}(0)$ from Bernstein and Lee's quoted result for $f(0)$.
[c] Our own calculation. We have not pursued the difference between our results for $f_{\rm II}(0)$ and those of Bernstein and Lee and of Meyer and Schiff because of the inherent ambiguity in the constant term in $f_{\rm II}(0)$.
[d] The substitution $\ln(\Lambda^2/M^2)\rightarrow -\ln\alpha$ has been made.
[e] See Ref. 8.

[5] The existence of $G(\infty)$ is basic to the method of Refs. 3 and 4, but its magnitude is not computable by the theory.

tained. The ξ-limiting process can be regarded as a generalization of the gauge-invariant Pauli-Villars regularization method: This prescription is known to suffer from a lack of uniqueness concerning the constant term in $f_{\mathrm{II}}(0)$.[6] The finite constant terms cancel out of the differences $f_{\nu_e}(0)-f_{\nu_\mu}(0)$ and $f(q^2)-f(0)$, so that these differences are of higher accuracy than $f(q^2)$ itself.

The W-meson contribution is, in the ξ-limiting theory, $f_{\mathrm{II}}(0)=6.4$ or 7.0, independent of any of the masses. It is, however, not yet clear whether the W meson exists: If it does, its mass must be large compared with the lepton masses. The W-meson contribution is thus, at least in the case of ν_e, small compared with the charged lepton contribution, and in any case is left somewhat ambiguous by the ξ-limiting calculation.

We have therefore chosen to omit the W-meson contribution entirely and to determine the neutrino form factor by a gauge-invariant F theory, using the W mass only as a cutoff. Our results are, therefore, applicable if the W meson does not exist or if, because of its large mass, it contributes inappreciably to the neutrino structure. We do not consider the F theory necessarily to be the infinite mass limit of the W theory.[7]

THE NEUTRINO FORM FACTOR IN THE F THEORY

The neutrino form factor in the F theory is now obtained directly from the calculation of the same lepton in quantum electrodynamics, being careful only to maintain gauge invariance. The relevant interaction Lagrangians are given by

Electromagnetic interaction:

$$L_{\mathrm{int}}{}^{\mathrm{EM}}=-ie\bar{\psi}_l\gamma_\mu\psi_l A_\mu\,, \tag{5}$$

weak interaction:

$$L_{\mathrm{int}}{}^{\mathrm{WEAK}}=-(iG/\sqrt{2})\{\bar{\psi}_l\gamma_\mu(1+\gamma_5)\psi_l\}\times\{\bar{\psi}_\nu\gamma_\mu(1+\gamma_5)\psi_\nu\}+\mathrm{H.c.}\,, \tag{6}$$

after a Fierz rearrangement. The matrix element for the diagram in Fig. 2 is then

$$i\{\bar{\nu}(p')\gamma_\nu(1+\gamma_5)\nu(p)\}\Pi_{\nu\mu}(q^2)\,, \tag{7}$$

where

$$\begin{aligned}\Pi_{\mu\nu}(q^2)=\Pi_{\nu\mu}(q^2)&=-\frac{G}{\sqrt{2}}\frac{ie}{(2\pi)^4}\int\mathrm{Tr}\{\gamma_\nu(1+\gamma_5)S(k)\gamma_\mu S(k-q)-\gamma_\nu(1+\gamma_5)S(k)\gamma_\mu S(k)\}d^4k\\&=-\frac{G}{\sqrt{2}}\frac{ie}{(2\pi)^4}\int\mathrm{Tr}\{\gamma_\nu S(k)\gamma_\mu S(k-q)-\gamma_\nu S(k)\gamma_\mu S(k)\}d^4k\end{aligned} \tag{8}$$

$$S(k)=\frac{\not{k}+im_l}{k^2+m_l^2}\,. \tag{9}$$

The expression (9) shows that only the electron or muon vector current contributions to $\Pi_{\mu\nu}(q^2)$. The parity violation effect is governed by the neutrino at the four-point weak vertex. The polarization tensor $\Pi_{\mu\nu}(q^2)$ in Eq. (9) is thus, if we make the substitution $Ge/\sqrt{2}\rightarrow e^2$, exactly the familiar photon vacuum polarization tensor in quantum electrodynamics. Thus, without further ado,[8]

$$\Pi_{\mu\nu}(q^2)=(q_\mu q_\nu-q^2\delta_{\mu\nu})C(q^2)\,, \tag{10}$$

where, in lowest order perturbation theory,

$$\begin{aligned}C(q^2)&=-\frac{1}{3q^2}\Pi_{\mu\mu}(q^2)\\&=\frac{G}{\sqrt{2}}\frac{e}{16\pi^2}\left\{-\tfrac{4}{3}\ln\left(\frac{\Lambda^2}{m_l^2}\right)-\frac{18}{9}+\tfrac{4}{3}\left[\frac{1}{\rho}+\left(1-\frac{1}{2\rho}\right)\left(1+\frac{1}{\rho}\right)^{1/2}\ln\left(\frac{(1+1/\rho)^{1/2}+1}{(1+1/\rho)^{1/2}-1}\right)\right]\right\}\,.\end{aligned} \tag{11}$$

$$\rho=q^2/4m_l^2\,. \tag{12}$$

Substituting (10)–(12) into (7), and then comparing with (3), we obtain

$$F(q^2)=-q^2C(q^2)\equiv-\frac{G}{\sqrt{2}}\frac{e}{16\pi^2}q^2f(q^2)\,, \tag{13}$$

$$f(q^2)=-\tfrac{4}{3}\ln\left(\frac{\Lambda^2}{m_l^2}\right)-\frac{18}{9}+\tfrac{4}{3}\left\{\frac{1}{\rho}+\left(1-\frac{1}{2\rho}\right)(1+1/\rho)^{1/2}\ln\left(\frac{(1+1/\rho)^{1/2}+1}{(1+1/\rho)^{1/2}-1}\right)\right\}\,, \tag{14}$$

[6] W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949). N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience Publishers, Inc., New York, 1959).

[7] G. Feinberg and A. Pais, Phys. Rev. **133**, B477 (1964).

[8] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company Inc., Reading, Massachusetts, 1959), Chap. 9. Note that the right-hand side of Eqs. (9–66) and (A5–27) should have an over-all negative sign.

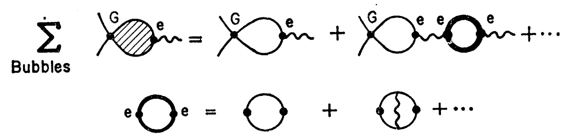

FIG. 3. The bubble diagrams which are of arbitrary order in e, but only first order in G. The virtual particles at the G vertex must be electrons or muons, but the bold-face bubble receives contributions from all charged particles.

where the Dirac equation for the neutrino has been used. The charge neutrality condition (4) appears in Eq. (13) as a consequence of gauge invariance. The expressions (10) and (11) are also gauge invariant. From Eq. (14), both $f(0)$ and $f(q^2)-f(0)$ can be easily derived; the results are given in the last column of Table I. Our expression for $f(q^2)-f(0)$ is identical with that of Bernstein and Lee [their Eqs. (9) and (10)] $(M^2 \gg m_l^2, q^2)$.

It is now clear that the ν_e charge radius $\langle r^2 \rangle = 6C(0)$ measures $Z_{3e}^{-1} = 1-(\sqrt{2}e/G)C(0)$ the principal (electron) part of the lowest order perturbation theoretic-calculation of Z_3^{-1}, the charge renormalization in quantum electrodynamics. If we consider, furthermore, all orders in electromagnetic radiative corrections, but only the first order in G, we sum over all the bubble diagrams (Fig. 3). The result is[9] then to replace Ge by $GeZ_3^{1/2} = Ge_0$, i.e., to renormalize the electron charge wherever it appears.

Parenthetically, we can comment on the higher order weak interaction effects, restricting ourselves to lowest order in e but considering diagrams of arbitrary order in G, i.e., the chain diagrams in Fig. 4. For a fixed value of G, or in the small momentum transfer region, applying the arguments of Landau *et al.*[10] or Feinberg and Pais,[7] we conclude that $f(q^2)$ vanishes in the limit of infinite cutoff Λ. In this respect the results of F theory and of W theory are very different.

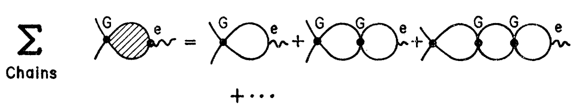

FIG. 4. The chain diagrams which are of arbitrary order in G, but only first order in e.

[9] M. A. Ruderman, University of California Laboratory Report UCRL-10336, 1962 (unpublished).

CONCLUSIONS

Summing up, we conclude that: (i) The qualitative observation of the ν_e and ν_μ electromagnetic form factors, while not bearing strongly on the questions of the existence of W mesons and the validity of ξ-limiting process, would indicate the neutrino-charged lepton couplings that are operative; (ii) the difference between the ν_e and ν_μ form factors or the expression $f(q^2)-f(0)$ is independent of W structure and ξ-limiting process; (iii) should the W meson not exist, a measurement of the ν_e charge radius determines the electron part of the charge renormalization $Z_{3e}^{1/2}$, as given in conventional quantum electrodynamics. The answers to the qualitative questions of the existence of neutrino form factors and of a difference between them, may be obtained sooner than an exact measurement of the momentum dependence of the neutrino form factors. Thus, far from discouraging the neutrino form factor experiments, the present note is intended to emphasize just what is likely to be learned from them.

Note added in proof. After this work was submitted for publication we learned that Ya. B. Zel'dovich and A. M. Perelomov, Zh. Eksperim. i Teor. Fiz. **39**, 1115 (1960) [English transl.: Soviet Phys.—JETP **12**, 777 (1960)] have also referred the neutrino form-factor calculation to the vacuum polarization tensor in quantum electrodynamics. Our results for the neutrino charge radius are, however, not exactly the same: Our result $C(0) = (Ge/\sqrt{2}4\pi)(1/3\pi)[-\ln(\Lambda^2/m^2)+\frac{1}{6}]$ (obtained by invariant cutoff) differs in the sign of $\frac{1}{6}$ from their formulas. The principal point is that the ν_e charge radius *is* inherently ambiguous but is related by $\langle r^2 \rangle = -6(G/\sqrt{2}e)(Z_{3e}^{-1}-1)$, where $G/\sqrt{2}e = (1.01\times 10^{-16}\ \mathrm{cm})^2$, to the perturbation-theory charge renormalization. This idea is apparently suggested already in Ref. 9.

[10] A. Abrikosov, A. Galanin, L. Gorkov, L. Landau, I. Pomeranchuk, and K. Ter-Martirosyan, Phys. Rev. **111**, 321 (1958).